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NOTE ON PROBLEM 118.

BY W. H. CARTER, VICE PRESIDENT CENTENARY COLLEGE, JACKSON, LA.

The solution of this problem published on page 241 of the October (1899) number is somewhat unsatisfactory as it involves the Differential Calculus.

I use the figure given. Pass a circle through the points D and C, and tangent to the line FH. Let E be the point of tangency. Then E is the required point. For at any other point of FH, as M, the $\angle CMD$ has a numerical measure equivalent to one-half of the difference between those of the arc CD and the other arc intercepted by its sides, while at E, the $\angle CED$ has the same measure as one-half the arc CD. The visual angle is therefore a maximum at E. The calculation of the distance FE is then easy.

Note.—Prof. J. W. Scroggs, of Eureka, Kas., sent in a neat and brief construction of problem 118, but it came too late for credit in the proper issue.

CALCULUS.

93. Proposed by JOHN R. JEFFREY, Student in Ohio State University, Columbus, Ohio.

Solve the following differential equation :

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=2x, \text{ when } x<1.$$

I. Solution by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

Let $x=\sin\theta$, then $d\theta/dx=\sec\theta$.

$$\frac{d\theta}{dx}=\frac{dy}{d\theta}\cdot\frac{d\theta}{dx}=\sec\theta\frac{dy}{d\theta}.$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\sec\theta\frac{dy}{d\theta}\right)=\frac{d}{d\theta}\left(\sec\theta\frac{dy}{d\theta}\right)\frac{d\theta}{dx}=\sec^2\theta\tan\theta\frac{dy}{d\theta}+\sec^2\theta\frac{d^2y}{d\theta^2}.$$

Substituting, our equation becomes

$$\frac{d^2y}{d\theta^2}+y=2\sin\theta\ldots\ldots (1).$$

Consider $\frac{d^2y}{d\theta^2}+y=0$.

$y=\cos\theta$ is a particular solution. Now let $y=z\cos\theta$......(2). Differentiating we get

$$\frac{d^2y}{d\theta^2}=\cos\theta\frac{d^2z}{d\theta^2}-2\sin\theta\frac{dz}{d\theta}-z\cos\theta.$$

Substitute this, and y from (2), in (1), and we get

$$\frac{d^2z}{d\theta^2}-2\tan\theta\frac{dz}{d\theta}=z\tan\theta\,.....\;(3).$$

Let $p=dz/d\theta$. Then (3) becomes

$$\frac{dp}{d\theta}-2\tan\theta.p=z\tan\theta\;.....(4).$$

Consider now $\frac{dp}{d\theta}-2\tan\theta.p=0$. Its solution is $p\cos^2\theta=c......(5)$.

Differentiate this, regarding c as variable, and we get

$$\frac{dp}{d\theta}=2\tan\theta.p+\frac{1}{\cos^2\theta}\frac{dc}{d\theta}.$$

This in (4) gives $\frac{dc}{d\theta}=\sin2\theta......(6)$.

$\therefore\; c=-\frac{1}{2}\cos2\theta+k_1$. Substitute this value of c in (5) and we get

$$\frac{dz}{d\theta}=-1+\tfrac{1}{2}(1+2k_1)\sec^2\theta.$$

$\therefore\; z=-\theta+\frac{1}{2}(1+2k)\tan\theta+k_2$. This value of z in (2) gives

$$y=-\theta\cos\theta+\tfrac{1}{2}(1+2k)\sin\theta+k_2\cos\theta.$$

Substitute for θ and we get

$$y=-\sin^{-1}x.\sqrt{(1-x^2)}+\tfrac{1}{2}x(1+2k)+k_2\sqrt{(1-x^2)},$$

or in a more general form

$$y=Ax+B\sqrt{(1-x^2)}-\sqrt{(1-x^2)}\sin^{-1}x,$$

which is the complete primitive.

II. Solution by **G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.**

Let $y=vx$, then the equation becomes

$$x(1-x^2)\frac{d^2v}{dx^2}+(2-3x^2)\frac{dv}{dx}=2x.$$

$$\therefore\; x^2(1-x^2)^{\frac{1}{2}}\frac{dv}{dx}=\sin^{-1}x-x\sqrt{(1-x^2)}+A.$$

$$\therefore\; \frac{dv}{dx}=\frac{\sin^{-1}x}{x^2(1-x^2)^{\frac{1}{2}}}+\frac{A}{x^2(1-x^2)^{\frac{1}{2}}}-1/x.$$

$$\therefore\ vx=Bx-(A+\sin^{-1}x)\sqrt{(1-x^2)}.$$

$$\therefore\ y=Bx-(A+\sin^{-1}x)\sqrt{(1-x^2)}.$$

III. **Solution by M. C. STEVENS, M. A., Professor of Mathematics, Purdue University, Lafayette, Ind., and GEORGE LILLEY, Ph.D., LL. D., Professor of Mathematics, University of Oregon, Eugene, Oregon.**

This equation is exact and the first integral is

$$(1-x^2)\frac{dy}{dx}+xy=x^2+c_1,\text{ or }\frac{dy}{dx}+\frac{x}{1-x^2}y=\frac{x^2}{1-x^2}+\frac{c_1}{1-x^2}.$$

This is a linear equation in which

$$\int Pdx=\int\frac{xdx}{1-x^2}=\log\frac{1}{\sqrt{(1-x^2)}}.$$

$$\therefore\frac{1}{\sqrt{(1-x^2)}}y=\int\frac{x^2dx}{(1-x^2)^{\frac{3}{2}}}+c_1\int\frac{dx}{(1-x^2)^{\frac{3}{2}}}.\quad\text{Whence } x<1.$$

$$\frac{1}{\sqrt{(1-x)^2}}y=\frac{x}{\sqrt{(1-x^2)}}-\sin^{-1}x+\frac{c_1x}{\sqrt{(1-x^2)}}+c_2,$$

or $y=cx+\sqrt{(1-x^2)}(c_2-\sin^{-1}x)$ when c is written for c_1+1.

This example is found on page 144, *Johnson's Differential Equations*.

IV. **Solution by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.**

First solve $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$.

Putting $\frac{dy}{dx}=p$, $(1-x^2)\frac{dp}{dx}-px+y=0$.

Differentiating, $(1-x^2)\frac{d^2p}{dx^2}-3x\frac{dp}{dx}=0$.

Putting q for $\frac{dp}{dx}$, $(1-x^2)\frac{dq}{dx}-3xq=0$.

Then $q=A(1-x^2)^{\frac{3}{2}}$, $p=\frac{Ax}{\sqrt{(1-x^2)}}+B$, $y=-A(1-x^2)^{\frac{1}{2}}+Bx$.

The particular integral may be found by the method of variation of parameters. Considering A and B variable, and substituting in the given equation,

$$-(1-x^2)^{\frac{3}{2}}\frac{d^2A}{dx^2}+(1-x^2)x\frac{d^2B}{dx^2}+x(1-x^2)^{\frac{1}{2}}\frac{dA}{dx}-x^2\frac{dB}{dx}=2x.$$

We may assume

$$x(1-x^2)^{\frac{1}{2}}\frac{dA}{dx}-x^2\frac{dB}{dx}=0.$$

Differentiating this and eliminating the second derivatives,

$$x(1-x^2)^{\frac{1}{2}}\frac{dA}{dx}+(1-x^2)\frac{dB}{dx}=-2x.$$

Solving for $\frac{dA}{dx}$ and $\frac{dB}{dx}$,

$$\frac{dB}{dx}=-2x,\ \frac{dA}{dx}=\frac{-2x^2}{(1-x^2)^{\frac{1}{2}}}.$$

Then $B=-x^2$, $A=\sin^{-1}x+x\surd(1-x^2)$.

The complete solution is then

$$y=(\sin^{-1}x-a)\surd(1-x^2)+(b-1)x,$$

a and b being arbitrary.

94. Proposed by **ALOIS F. KOVARIK**, Instructor in Mathematics and Science, Decorah Institute, Decorah, Iowa.

Find the minimum isosceles triangle that can be described about a given ellipse, having its base parallel to the major axis. [Ex. 16, page 166, *Rice and Johnson's Differential Calculus*.]

Solution by **GEORGE R. DEAN**, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

The equation of the tangent to an ellipse in terms of the slope being

$$y=mx+\surd(a^2m^2+b^2),$$

the altitude of the triangle will be the intercept of the tangent on the y-axis plus the semi-minor axis or $b+\surd(a^2m^2+b^2)$.

The base of the triangle found by putting $y=-b$ in the equation of the tangent is

$$\frac{-b-\surd(a^2m^2+b^2)}{m}.$$

The function of which a minimum value is to be found is accordingly

$$\frac{[b+\surd(a^2m^2+b^2)]^2}{m}.$$

The derivative of this function equated to zero gives after reduction

$$a^4m^4-3a^2b^2m^2=0.$$

The value of m is $\pm b/a\surd3$, and $a^2m^2+b^2=4b^2$.

The intercept on the y-axis is therefore $2b$, and the altitude of the triangle is $3b$.